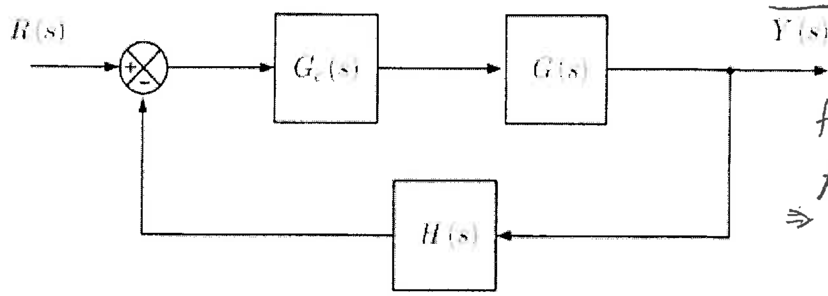
SOLUTION

$$\frac{G_o H(0) K_p}{\left|1 + \frac{j\omega_c}{Q\omega_o} + \left(\frac{j\omega_c}{\omega_o}\right)^2\right|} = \frac{G_o H(0) K_p}{\sqrt{\left[1 - \left(\frac{f_c}{f_o}\right)^2\right]^2 + \left(\frac{f_c}{Q f_o}\right)^2}} = 1$$

Questions (22) to (25) considers the following system:

$$K_p = \frac{\sqrt{\left[1-\left(\frac{f_c}{f_o}\right)^2\right]^2 + \left(\frac{f_c}{Q f_o}\right)^2}}{G_o H(0)}$$



$Q = 1.6$ $\quad G_o = 50$

$f_o = 1{,}000$ $\quad H(0) = \frac{1}{2}$

$f_c = 1500$

$\Rightarrow K_p = 0.0617$

where the plant $G(s) = \frac{G_o}{1+\frac{s}{Q\omega_o}+\left(\frac{s}{\omega_o}\right)^2}$, $G_o = 50$, $Q = 1.66$, $\omega_o = 2\pi(1000)$ and $H(s) = \frac{1}{2}$.

Transfer function $G_c(s)$ represents the compensator. (Hint: $10^{\frac{1}{(2*1.66)}} = 2$)

$f_o = 1000$

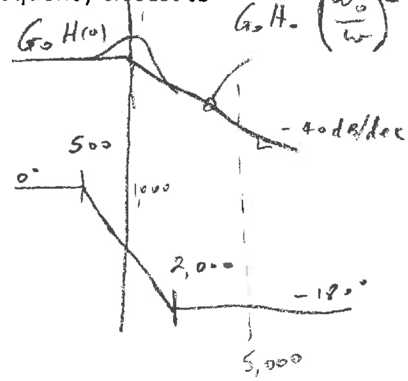
22. The uncompensated loop gain (i.e. $G_c(s) = 1$) has a unity gain frequency closest to
    a. 2 kHz
    b. 3 Khz
    c. 4 Khz
    d. 5 khz
    e. 6 kHz

$G_o H_o \left(\frac{\omega_o}{\omega}\right)^2 \Rightarrow 25\left(\frac{1000}{f}\right)^2$

$25 \times 10^6 = f_c^2$

$\Rightarrow f_c = 1000\sqrt{25}$

$= 5{,}000$



23. The phase margin of the uncompensated system is closest to
    a. 60 degrees
    b. 40 degrees
    c. 20 degrees
    d. 0 degrees
    e. -20 degrees

PHASE @ 5kHz = -180°

$\Rightarrow$ PM = 180° - 180°

= 0°

24. We would like to design a proportional compensator with gain K_p. What should K_p be to achieve a unity gain bandwidth of 1.5 kHz.
    a. $K_p = 0.03$
    b. $K_p = 0.05$
    c. $K_p = 0.07$
    d. $K_p = 0.09$
    e. $K_p = 0.10$

NOT CORRECT

$G_o H(0) K_p \left(\frac{f_o}{f_c}\right)^2 = 1 \Rightarrow K_p = \left(\frac{f_c}{f_o}\right)^2 \frac{1}{G_o H(0)}$

$K_p = \left(\frac{1{,}500}{1{,}000}\right)^2 \frac{1}{25} = 0.0900$

$\Rightarrow$ INACCURATE

SEE CORRECT SOLUTION ABOVE

25. The phase margin achieved by the proportional compensator design of Question 24 is closest to:
    a. 30 degrees
    b. 50 degrees
    c. 70 degrees
    d. 90 degrees

Phase @ $f_c = 1.5$ kHz $= -\tan^{-1}\left[\frac{\frac{1}{Q}\left(\frac{f_c}{f_o}\right)}{1-\left(\frac{f_c}{f_o}\right)^2}\right]$

$Q = 1.66$

$f_o = 1{,}000$

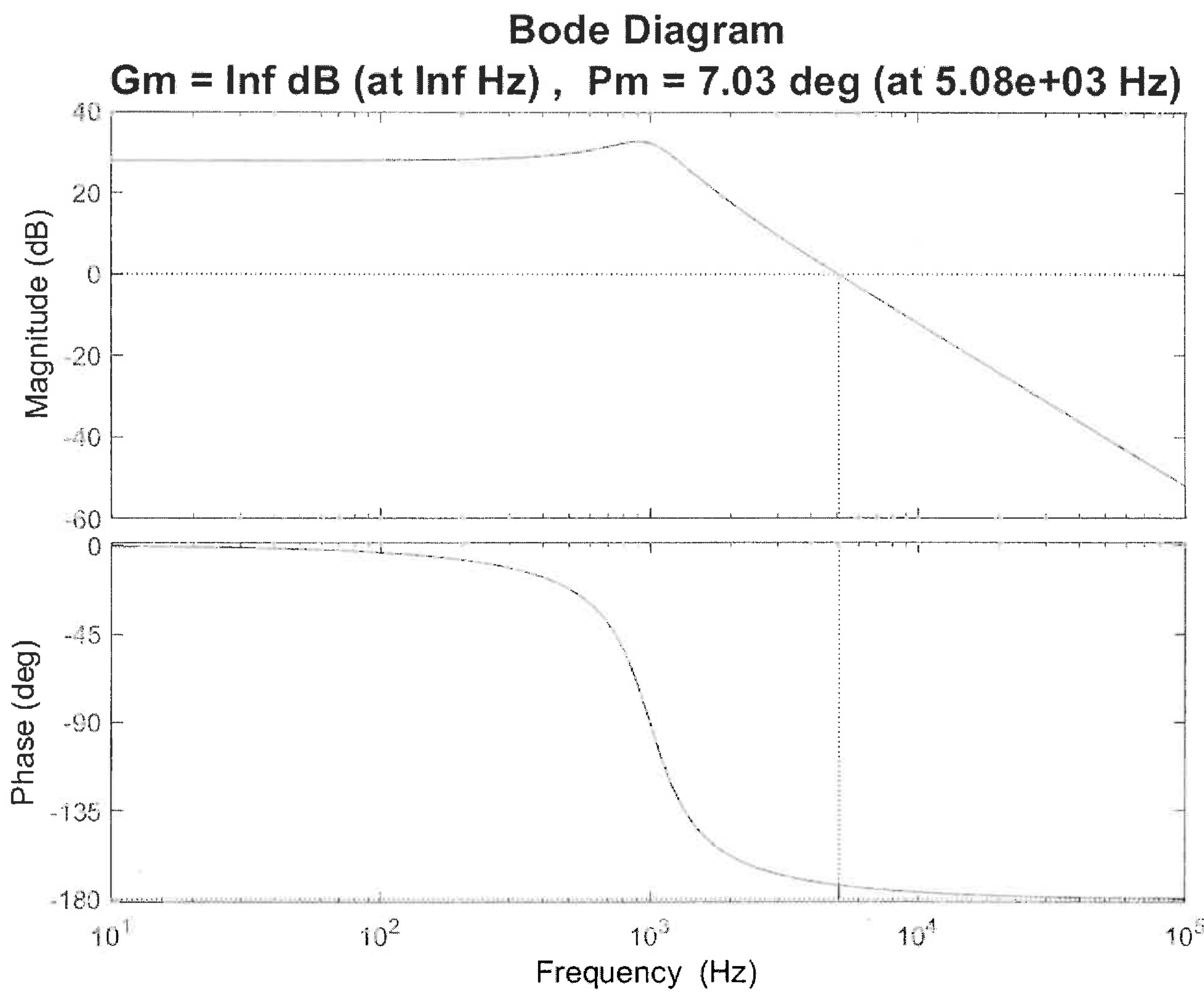
$f_c = 1{,}500$

$\Big\}$ PHASE = -144.1372°

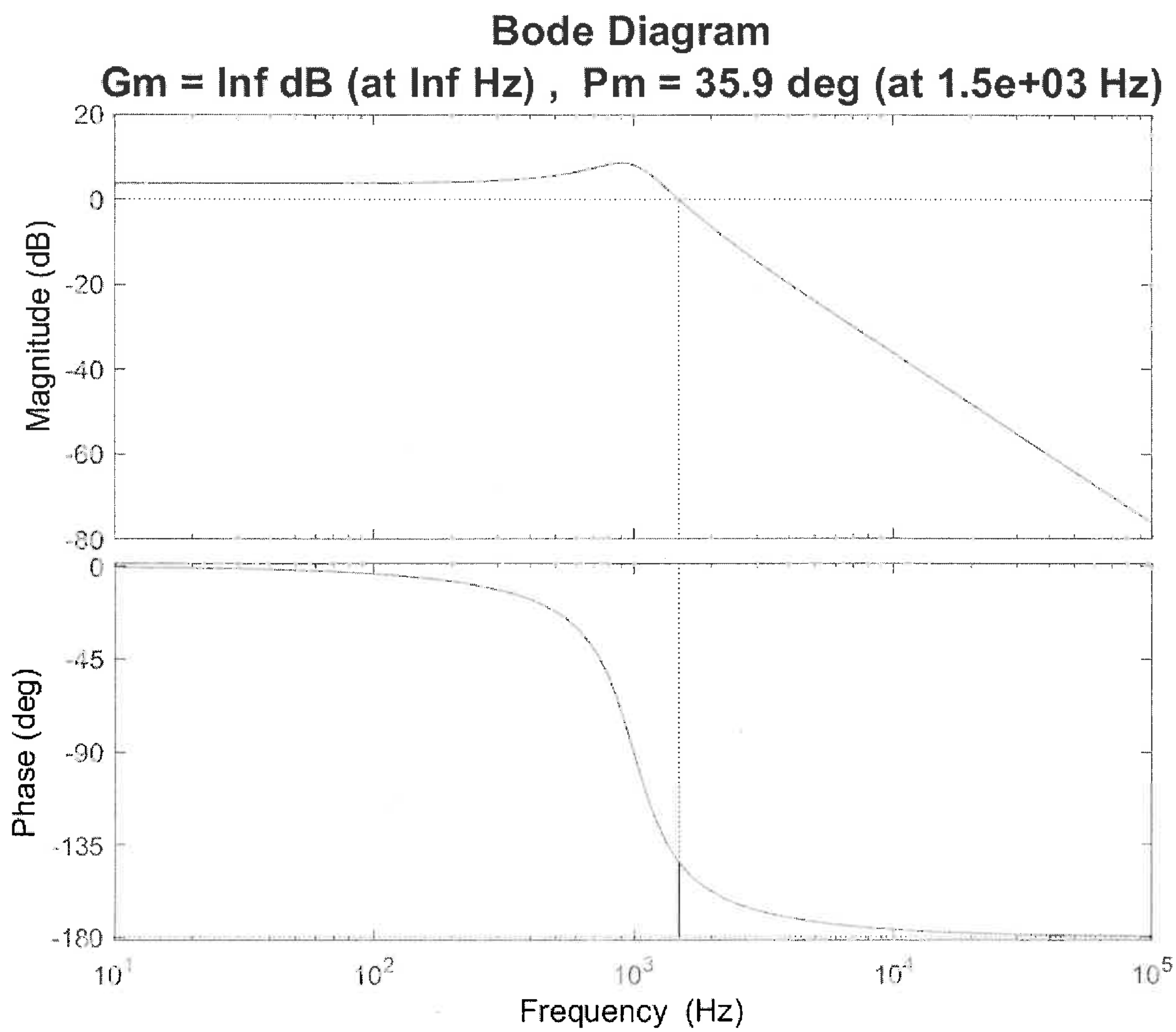
$\Rightarrow$ PM = 180° - 144.1372°

= 35.86°

EXACT UNCOMPENSATED LOOP GAIN

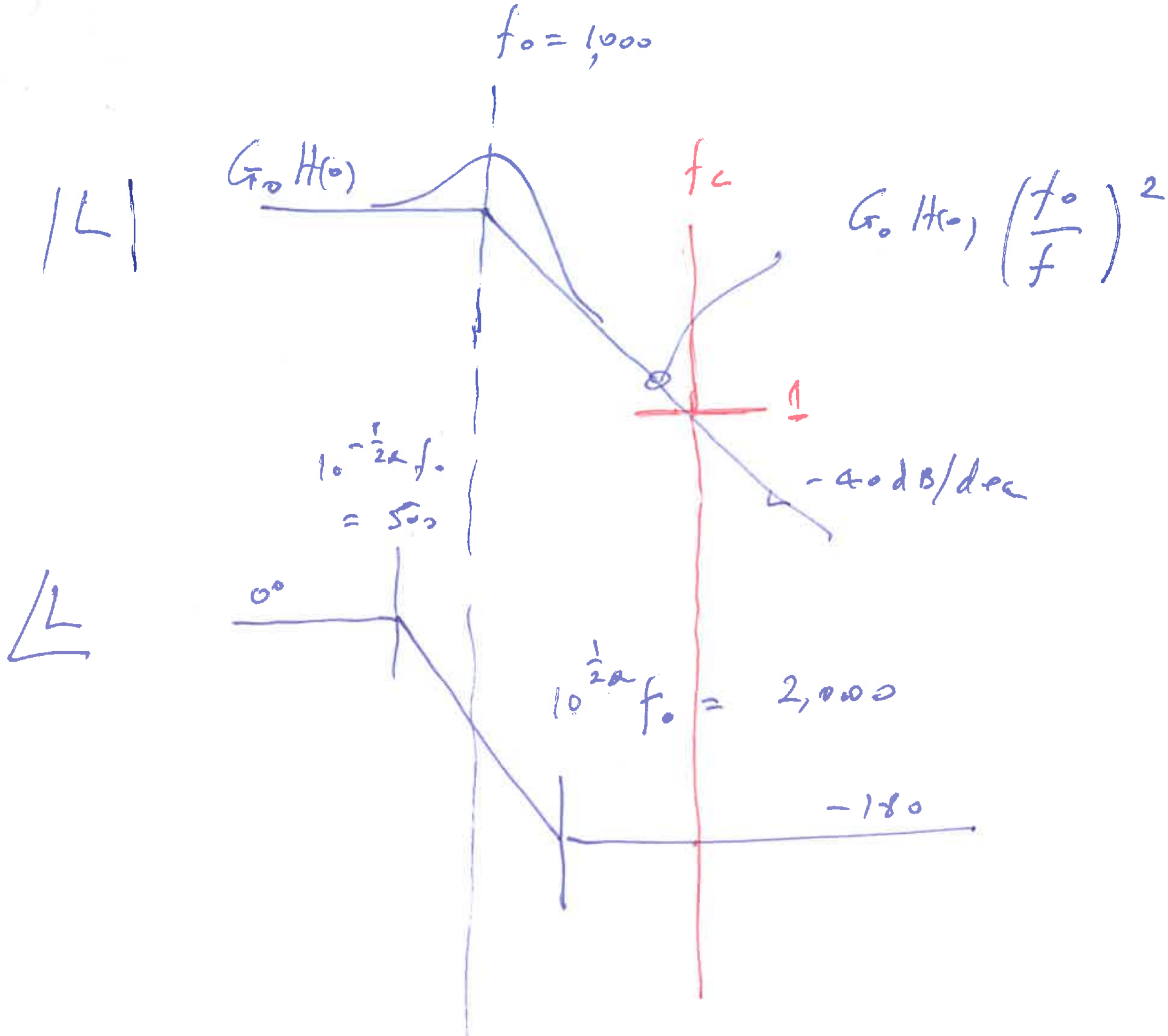


EXACT COMPENSATED LOOP GAIN WITH PROPORTIONAL COMPENSATOR



```
clear
close all
format compact

% Question 24 on ECE317 Final needs to be calculated accurately to get the
% right answer

wc = 2*pi*1500
wo = 2*pi*1000
H = 0.5
Go = 50
Q = 1.66

T0 = Go*H
s = tf('s')

% uncompensated
Tuncomp = T0/(1+s/(Q*wo)+(s/wo)^2)
figure(1)
margin(Tuncomp)
h = gcr;
h.AxesGrid.Xunits = 'Hz';
h.AxesGrid.TitleStyle.FontSize = 16;
h.AxesGrid.XLabelStyle.FontSize = 12;
h.AxesGrid.YLabelStyle.FontSize = 12


%%% Find compensation gain Kp
x = wc/wo
Kp = sqrt( (1-x^2)^2 + (1/Q*x)^2 ) /(H*Go) % this is the answer to Q. 24
% Kp = sqrt( (1-(wc/wo)^2)^2 + (1/Q*(wc/wo))^2 ) /(H*Go)

phase_margin = 180-atan2d(1/Q* x, 1-x^2)  % this is the answer to Q. 25
% phase_margin = 180-atan2d(1/Q* wc/wo, 1-(wc/wo)^2)

%%% Compensated loop gain
Tcomp = Kp*Tuncomp
figure(2)
margin(Tcomp)
h = gcr;
h.AxesGrid.Xunits = 'Hz';
h.AxesGrid.TitleStyle.FontSize = 16;
h.AxesGrid.XLabelStyle.FontSize = 12;
h.AxesGrid.YLabelStyle.FontSize = 12
```

DEC. 5th



22

$$G_0 H(0) \left(\frac{f_0}{f_c}\right)^2 = 1 \Rightarrow f_c = f_0 \sqrt{G_0 H(0)}$$

$$= 1000 \times 5$$

$$= 5\ kHz.$$

$\Rightarrow$ d

23

Phase @ $f_c = -180°$

$\Rightarrow PM = 180° - 180° = 0°.$

$\Rightarrow$ d

24

$$L_{comp}(s) = \frac{K_p G_o H(0)}{1 + \frac{s}{Q\omega_o} + \left(\frac{s}{\omega_o}\right)^2}$$

$$\left|L_{comp}(s)\right|_{f=f_c} = \frac{K_p G_o H(0)}{\sqrt{\left(1 - \left(\frac{f_c}{f_o}\right)^2\right)^2 + \left(\frac{f_c}{Q f_o}\right)^2}} = 1$$

$s = j\omega$

$\omega = 2\pi f$

$$\boxed{\begin{aligned} z &= a + jb \\ |z| &= \sqrt{a^2 + b^2} \end{aligned}}$$

$$\Rightarrow \quad K_p = \frac{\sqrt{\left(1 - \left(\frac{f_c}{f_o}\right)^2\right)^2 + \left(\frac{f_c}{Q f_o}\right)^2}}{G_o H(0)}$$

$$= \underline{\underline{0.0619}} \Rightarrow \text{C}$$

$Q = 1.66$

$f_o = 1{,}000$

$f_c = 1{,}500$

$G_o = 50$

$H(0) = 0.5$

③

25

Phase @ $f_c = 1{,}500$ Hz

$$= -\tan^{-1}\left[\frac{\frac{1}{\alpha}\frac{f_c}{f_o}}{1-\left(\frac{f_c}{f_o}\right)^2}\right]$$

$\alpha = 1.6$

$f_o = 1{,}000$

$f_c = 1{,}500$

$= -144.1372$

$\Rightarrow$ $P_m = 180^\circ - 144.1372$

$= \underline{35.86^\circ}$

$\Rightarrow$ a